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Rolling tachyon

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Abstract: We discuss construction of classical time dependent solutions in open string (field) theory, describing the motion of the tachyon on unstable D-branes. Despite the fact that the string field theory action contains infinite number of time derivatives, and hence it is not a priori clear how to set up the initial value problem, the theory contains a family of time dependent solutions characterized by the initial position and velocity of the tachyon field. We write down the world-sheet action of the boundary conformal field theories associated with these solutions and study the corresponding boundary states. For D-branes in bosonic string theory, the energy momentum tensor of the system evolves asymptotically towards a finite limit if we push the tachyon in the direction in which the potential has a local minimum, but hits a singularity if we push it in the direction where the potential is unbounded from below.

Keywords: Tachyon Condensation, D-branes, String Field Theory.

Contents

1. Introduction

Much work has been devoted to the study of tachyon potential and various classical solutions in string field theory on an unstable D-brane system [1, 2]. However most of the solutions studied so far have been time independent solutions representing either the tachyon vacuum or static D-branes of lower dimension. Recently Gutperle and Strominger considered the process of production and decay of unstable branes, and proposed that the corresponding solution can be described as a space-like D-brane [3].

There are many issues associated with the decay of unstable branes in string theory. We shall study the classical decay process, namely, what happens if we displace the tachyon a little bit away from its maximum and let it roll according to its equations of motion. Although this is a natural question in any scalar field theory around the maximum of the potential, it is not obvious if this is a sensible question in string field theory. The string field theory action contains infinite number of time derivatives, and hence *a priori* the initial value problem is not well-defined. Nevertheless, we shall show that it is possible to construct a family of classical solutions of the string field theory equations of motion characterized by the initial position and velocity of the tachyon field. The construction is similar in spirit to that of [3] in that we get it by Wick rotation of a known solution, but the details are very different.

Another issue in the study of tachyon condensation is the nature of the time evolved configuration. In studying the classical decay of an unstable soliton in a field theory (e.g. domain wall in ϕ^3 theory [4]), one can displace the field configuration from its original value and let it evolve in time. For simplicity we can restrict to deformations which do not depend on the coordinates along the brane, — in terms of fields living on the brane this corresponds to constant field configuration. Typically in such a case the tachyonic field on the brane couples to various other fields, including those which are not localised on the brane and hence describes bulk modes. As a result even if the initial configuration contains only tachyonic deformation, with the energy density stored fully inside the tachyon field, as the system evolves in time the energy gets distributed among all the modes, including the infinite number of fields on the brane describing the bulk modes. In a generic case, if we wait long enough then eventually most of the energy is transfered to the bulk modes, and we can interprete this process as the result of the decay of the unstable brane into classical radiation.

In open string field theory, this issue is much more complicated. Unlike in an ordinary field theory, in this case we do not expect any conventional bulk modes, since there are no physical open string modes away from the original D-brane [5, 6, 7]. Nevertheless one might expect that there are non-Fock space excitations of open string field configuration describing closed string excitations [8]–[11], and that the decay process transfers the energy of the original brane to these closed string excitations. We shall argue that this is not what happens in classical open string field theory. Instead, during the classical time evolution of the open string field describing the decay, the energy density resides in the plane of the original brane. This situation is similar to that in the case of toy models of tachyon condensation [12] where again there are no bulk modes and the energy density does not dissipate [15]. We also find that the energy momentum tensor of the system evolves smoothly towards an asymptotic configuration instead of undergoing oscillation about the minimum of some effective potential. It is not clear how to reconcile this with the result from boundary string field theory that the minimum of the tachyon potential is at a finite distance away from the maximum [13, 14].

One of the results that follows from the general analysis of this paper is that for D-branes in bosonic string theory, there is a qualitative difference between pushing the tachyon on the side of the maximum where there is a local minimum and the side where the potential is unbounded from below. In the boundary conformal field theory description, the former corresponds to adding a positive definite boundary term to the world-sheet action whereas the latter corresponds to adding a negative definite term. In the boundary state description, the former corresponds to a boundary state with smooth time evolution, while the latter hits a singularity at a finite time.

The rest of the paper is organised as follows. In section 2 we show the existence of a two parameter family of time dependent solutions, labelled by the initial position and velocity of the tachyon field, describing the rolling of the tachyon field on an unstable D-brane. In section 3 we construct the action of the boundary conformal field theory (BCFT) describing these solutions and in section 4 we explore the nature of the corresponding boundary states. In section 5 we study the decay process using an effective field theory. We conclude in section 6 with a few additional comments.

2. Time dependent solutions of open string field equations

In order to find a time dependent solution of the open string field equations, we can make a consistent truncation of the equations of motion restricting the fields to a universal subspace as follows. We shall assume that the original D-brane configuration is described by a unitary matter BCFT of central charge 25, together with a free scalar field representing the time coordinate X^0, and the ghost BCFT. Following the arguments of [2, 5] it is easy to show that if we restrict the open string field to the subspace of the full BCFT created from the vacuum by various operators associated with the scalar field X^0, the Virasoro generators of the $c = 25$ BCFT, and the ghost oscillators, then the equations of motion of the fields outside the subspace are automatically satisfied. Thus this provides a consistent truncation of the string field theory equations of motion, and we can look for the classical solutions describing time evolution of the brane within this subspace. Furthermore the action restricted to this subspace is universal, independent of the choice of the $c = 25$ BCFT. This shows that the solution describing the decay of an unstable D-brane is universal, independent of which D-brane or spatial background we begin with.

Let us first consider the linearized equations of motion of string field theory where we ignore the coupling between various fields. In this case we can treat the tachyon as an ordinary scalar field of negative mass^2. For definiteness let us focus on the bosonic string theory, where in $\alpha' = 1$ unit the tachyon has mass^2 $= -1$. In this case, for small deformations the solution for the tachyon field will have the form

$$T(x^0) = Ae^{x^0} + Be^{-x^0} , \tag{2.1}$$

where A and B are constants determined by the initial condition:

$$T(0) = \lambda , \qquad \left.\frac{\partial T}{\partial x^0}\right|_{x^0=0} = u . \tag{2.2}$$

We shall for simplicity take $u = 0$. Also, we shall take $\lambda > 0$, so that T rolls towards the extremum of the action representing the tachyon vacuum, which is at a positive value of T [1]. This gives

$$A = B = \frac{1}{2}\lambda . \tag{2.3}$$

Hence

$$T(x^0) = \lambda \cosh(x^0) . \tag{2.4}$$

As long as λ is small and x^0 is not too large, we expect the linearized approximation to be valid, and the above represents the expected evolution of the tachyon field. The question now is: can we modify the above solution in a systematic manner to take into account the effect of the interaction? We can begin by trying to construct a solution of the equations of motion:

$$Q_B|\Psi\rangle + |\Psi * \Psi\rangle = 0 , \tag{2.5}$$

as an expansion in λ:

$$|\Psi\rangle = \sum_n \lambda^n |\chi^{(n)}\rangle . \tag{2.6}$$

The leading term is given by:

$$|\chi^{(1)}\rangle = c_1|0\rangle_g \otimes \cosh(X^0(0))|0\rangle_m . \tag{2.7}$$

The subscripts g and m refer to ghost and matter sectors, respectively. We can construct the higher order terms in the solution in Siegel gauge

$$b_0|\Psi\rangle = 0\,, \tag{2.8}$$

by choosing:

$$|\chi^{(n)}\rangle = -\frac{b_0}{L_0}\sum_{m=1}^{n-1}\left|\chi^{(m)} * \chi^{(n-m)}\right\rangle. \tag{2.9}$$

This gives a way to construct the nth order term in terms of lower order terms. However this procedure breaks down if $b_0\sum_{m=1}^{n-1}|\chi^{(m)} * \chi^{(n-m)}\rangle$ contains a state with zero L_0 eigenvalue. This is related to the question of whether the matter sector vertex operator $\cosh(X^0)$ is exactly marginal. In this case, however, the marginality of this operator follows from the result in the Wick rotated theory where we replace X^0 by iX. X is now a free scalar field describing a unitary conformal field theory of $c = 1$. Under this rotation $\cosh(X^0)$ gets transformed to $\cos(X)$, and this is known to represent an exactly marginal deformation in this Wick rotated theory [16, 17, 18]. This shows that the original operator $\cosh(X^0)$ is also exactly marginal, and we do not encounter any obstruction in solving eq. (2.9) in a power series expansion in the parameter λ. (If the coordinate x were compactified on a circle of *critical radius* $R = 1$, the perturbation by $\cos(X)$ would create a single codimension 1 D-brane on the circle. However since the direction x is non-compact, this perturbation creates an array of codimension 1 D-branes in the Wick rotated theory, seperated by an interval of 2π.)

More explicitly we can proceed as follows. In the Wick rotated theory, the solution of the string field theory equations of motion for different values of the parameter λ can be constructed explicitly using level truncation [7]. We can simply take this solution and make an inverse Wick rotation $X \to -iX^0$ to generate a time dependent solution. One must of course make sure that Wick rotation of the solution preserves the reality condition, but this can be seen to be true as follows.[1] The solution of ref. [7] is invariant under the twist transformation (under which a string field component picks up a phase of $(-1)^l$ where l denotes the level of the oscillators used to create the associated state from the zero momentum tachyon state $c_1|0\rangle$) as well as the transformation $X \to -X$. Thus it can be expressed as linear combination of states, each of which only involves even total powers of the oscillator α_{-n} of X and momentum k along the X direction, and hence remains real under the inverse Wick rotation $k \to ik_0$, $\alpha \to -i\alpha^0$. Thus a real field configuration remains real after the inverse Wick rotation.

This shows the existence of a one parameter family of time dependent solutions, characterized by the marginal deformation parameter λ. A more general solution, labelled by two parameters, can be constructed by simply time-translating the original solution, and the two parameters can be interpreted as the initial position and velocity of the tachyon field. Thus we see that despite the non-locality of the string field theory action, it does admit time dependent solutions labelled by initial position and velocity of the tachyon

[1] See [19] for a detailed discussion on reality condition on string field.

field, as expected of an ordinary field theory with two derivative actions. However, since for all these solutions the tachyon comes to rest near the maximum of the potential at a certain instant of time, the total energy of each of these solutions is less than that of the unstable D-brane. Solutions for which the total energy is larger than that of the unstable D-brane can be constructed by beginning with $T(x^0) = u \sinh x^0$ and then iterating this using eq. (2.9). The marginality of the operator $\sinh(X^0)$ again follows from the marginality of the operator $\sin(X)$ in the Wick rotated theory. Since under Wick rotation $u \sinh x^0$ becomes $iu \sin x$, and hence T becomes imaginary, the time dependent solution is not related to any physical solution in the Wick rotated theory. But the marginality of the operator $\sin(X)$ [16, 17, 18] would guarantee the existence of a complex solution in the Wick rotated theory whose leading term involves $T(x) = iu\sin(x)$. Under inverse Wick rotation this gives a real solution in the original theory as a formal power series expansion in u.

Of course it is quite likely that all these solutions break down at sufficiently large x^0, since the higher order terms in the expansion proportional to $\cosh(nx^0)$ or $\sinh(nx^0)$ grow with x^0. What we can hope for is that the solution exists at least for a finite range of values of x^0 during which we can use it to follow the time evolution of the string field. Since the expansion of the solution is in powers of $\lambda \cosh x^0$ (or $u \sinh x^0$), a rough estimate of the region of convergence for small λ is $x^0 < \ln(1/|\lambda|)$ (or $x^0 < \ln(1/|u|)$). Beyond this limit we need to use other methods for studying the system, as will be discussed in sections 3 and 4. In this context we also note that even for $x^0 = 0$, the solution of [7] can be constructed only for a limited range of λ: $|\lambda| \le \bar{\lambda}$ with $\bar{\lambda} \simeq .45$. This amounts to a restriction on the initial displacement of the tachyon field from its equilibrium value.

Before we leave this topic let us remark that the solution obtained this way for a generic λ is not the inverse Wick rotated version of a codimension one lump solution representing a lower-dimensional D-brane. The codimension one lump representing a lower-dimensional D-brane appears for a specific value λ_c of the deformation parameter λ.[2] We shall argue in section 4 that this critical value λ_c corresponds to placing the tachyon at the minimum of its potential in the Wick rotated theory. For other values of λ, — notably for small λ, — the resulting conformal field theory is solvable [16, 17, 18], but cannot be interpreted as a D-brane of one lower dimension. This in turn shows that the time dependent solution for a generic λ should not be thought of as an inverse Wick rotated version of a lower-dimensional D-brane. Furthermore, even for $\lambda = \lambda_c$, the lump solution should be thought to be located not at $x = 0$, but at $x = \pi$ where $\lambda \cos(x)$ reaches its minimum. Thus the origin of the x^0 coordinate, where the tachyon begins rolling, does not map under Wick rotation to the location of the lump solution.

We could repeat this construction for describing the decay of an unstable brane or a brane anti-brane pair in type-II string theory. The arguments follow a similar line with the tachyon now having a mass^2 of $-1/2$ in the $\alpha' = 1$ unit, and hence represented by a solution of the form

$$T(x^0) = \lambda \cosh\left(\frac{x^0}{\sqrt{2}}\right), \tag{2.10}$$

[2]The relation between λ_c and $\bar{\lambda}$ has not beeen conclusively determined [7].

to first order in λ. In the Wick rotated theory the corresponding vertex operator is exactly marginal [20], which guarantees the existence of a solution in string field theory. The complete solution whose first order term is given by (2.10) has the property that the $\partial_0 T$ vanishes at $x^0 = 0$. This reflects the fact that at $x^0 = 0$, the tachyon field has zero velocity. On the other hand the Wick rotated version of the solution, for which $T(x) \sim \lambda \cos(x/\sqrt{2})$ for small λ, should be thought of as an anti-kink located at $x = \pi/\sqrt{2}$, and a kink located at $x = 3\pi/\sqrt{2}$, i.e. at the points on the circle where the tachyon field itself vanishes. In particular if we increase λ, then for an appropriate value λ_c of λ, it is at these points where the lower-dimensional D-branes are created. Thus we see again that Wick rotation $x^0 \to ix$ does not map the origin of the x^0 coordinate, where the tachyon begins rolling, to the location of the soliton solution.

Finally we note that both in the case of bosonic string theory and the superstring theory, we could get other solutions by inverse Wick rotation of the solutions describing lump / kink solutions on a circle of radius other than the critical radius where the deformation is marginal. The interpretation of these solutions is not entirely clear to us; however since at the critical radius they seem to represent the tachyon placed at its minimum, this may be the case for other radii as well.

3. Conformal field theory description

As pointed out already, the procedure outlined in the last section gives us a way to construct time dependent solution of the equations of motion for sufficiently small x^0, describing the initial stage of the brane decay, but does not really allow us to follow the time evolution for large x^0. Thus we need to invoke other methods to analyze the system for large x^0.

One such method would be the techniques of two-dimensional conformal field theory. As discussed already, the Wick rotated solution can be described by a solvable boundary conformal field theory [16, 17, 18], and hence one might try to describe the physics of the solution describing the rolling of the tachyon by analysing the inverse Wick rotation of this solvable conformal field theory. This is described by the world-sheet action:

$$-\frac{1}{2\pi}\int d^2z\partial_z X^0\partial_{\bar{z}} X^0 + \widetilde{\lambda}\int dt \cosh X^0(t)\,, \tag{3.1}$$

where t parametrizes the boundary of the world-sheet, and the deformation parameter $\widetilde{\lambda}$ of conformal field theory is related to the parameter λ appearing in string field theory solution via some functional relation. For small λ, $\widetilde{\lambda} \simeq \lambda$ [7, 21]. We note from eq. (3.1) that for small positive λ (and hence positive $\widetilde{\lambda}$) the boundary term gives a positive contribution to the action. In contrast if we had chosen λ to be negative, the boundary term will be negative, and could destabilize the theory. According to eq. (2.2) this corresponds to pushing the tachyon to the wrong side, where it can roll down all the way to $-\infty$. Presumably this is the instability that is seen in the conformal field theory description.[3]

[3]This qualitative difference between positive and negative values of $\widetilde{\lambda}$ is not visible in the Wick rotated theory [22].

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For solutions with total energy larger than the brane tension, the $\cosh(X^0(t))$ term in (3.1) is replaced by $\sinh(X^0(t))$ and we see an instability for either sign of $\widetilde{\lambda}$ since $\sinh(X^0(t))$ is unbounded from above and below. This is presumably related to the fact that in this case either in the past or in the future the tachyon rolls to the wrong side of the maximum where the potential is unbounded from below.

In contrast, for an unstable D-brane in superstring theory the solution describing the rolling of the tachyon is described by a boundary perturbation of the form [20]:

$$\widetilde{\lambda} \int dt\, \sigma_1 \otimes \psi^0(t) \sinh X^0(t) \,, \tag{3.2}$$

where σ_1 is the Chan-Paton factor and ψ^0 is the superpartner of X^0. Thus both signs of $\widetilde{\lambda}$ are allowed in this case.

We shall not analyze these theories here, but note that since the Wick rotated version of these theories are solvable [16, 17, 18, 20], these theories are likely to be solvable as well. We also note the following point. Since the $\widetilde{\lambda}$-deformation of the original conformal field theory involves only the coordinate X^0, the part of the boundary conformal field theory involving the space-like coordinates remains unchanged under this deformation. In particular the coordinates transverse to the original brane, carrying Dirichlet boundary condition, will continue to carry Dirichlet boundary condition. Thus we would expect that as the configuration evolves in time, the energy density of the system is confined to the plane of the original brane. This can be seen in particular by probing the system by closed string vertex operators; if we take a closed string vertex operator carrying a factor of $e^{ik_\perp . X_\perp}$, and compute its one point function on the disk with the boundary perturbation appropriate to the rolling solution, the answer is independent of $k_\perp$ due to the Dirichlet boundary condition on the coordinates $X_\perp$, or at most a polynomial in $k_\perp$ if the closed string vertex operator involves additional explicit factors of $k_\perp$ and/or $\partial X_\perp$. In position space this implies that the coupling of the closed strings to the system is proportional to $\delta(x_\perp)$ or its derivative, indicating that the time dependent configuration is still firmly localized on the surface $x_\perp = 0$. This is in contrast to the generic decay of a soliton in a quantum field theory, where the initial energy of the lump soliton is eventually transfered to the bulk modes which carry the energy away from the brane.

4. Boundary state

We shall now analyze the boundary state associated with the rolling tachyon solution to determine what kind of source it produces for closed strings. We shall restrict our analysis in this section to bosonic string theory.

We consider the rolling tachyon solution in open bosonic string theory associated with the world-sheet action (3.1). As usual we shall attempt to determine the associated boundary state by beginning with the boundary state in the Wick rotated theory and then making an inverse Wick rotation. The boundary state in the Wick rotated theory, described by a free non-compact scalar X with boundary perturbation $\widetilde{\lambda} \int dt \cos(X(t))$ was analysed in [17]. It is given by:

$$|\mathcal{B}\rangle \propto \sum_j \sum_{m=-j}^{j} D^j_{m,-m}(R)|j;m,m\rangle\rangle\,, \tag{4.1}$$

where the sum over j runs over $0, 1/2, 1, \ldots$, R is the SU(2) rotation matrix:

$$R = \begin{pmatrix} \cos(\pi\widetilde{\lambda}) & i\sin(\pi\widetilde{\lambda}) \\ i\sin(\pi\widetilde{\lambda}) & \cos(\pi\widetilde{\lambda}) \end{pmatrix}, \tag{4.2}$$

$D^j_{m,-m}(R)$ is the spin j representation matrix of this rotation in the J_z eigenbasis, and $|j;m,m\rangle\rangle$ is the *Virasoro* Ishibashi state [23] built over the primary $|j;m,m\rangle$ in the $c=1$ CFT with momentum $2m$ and conformal weight (j^2, j^2). This primary can be expressed in the form:

$$|j;m,m\rangle = e^{i\varepsilon(j,m)}\mathcal{O}_{j,m}e^{2imX(0)}|0\rangle_c\,, \tag{4.3}$$

where $e^{i\varepsilon(j,m)}$ is a phase factor, $\mathcal{O}_{j,m}$ is a combination of oscillators of total level $j^2 - m^2$ both in the holomorphic and the anti-holomorphic sector, and $|0\rangle_c$ is the SL$(2,\mathbb{C})$ invariant Fock vacuum for the closed string.

Since the expression is somewhat complicated, we shall focus on the part of the boundary state that does not involve any X oscillators. This requires picking up only the primary states $|j;j,j\rangle$ and $|j;-j,-j\rangle$ in eq. (4.1), whose expressions, according to eq. (4.3), does not involve any oscillators since $j^2 - m^2$ vanishes. Since

$$D^j_{j,-j} = D^j_{-j,j} = (i\sin(\widetilde{\lambda}\pi))^{2j}\,, \tag{4.4}$$

according to the convention of [17], the oscillator free part of the boundary state is given by:

$$|\mathcal{B}_0\rangle \propto \left[1 + \sum_{j\neq 0}\left(i\sin(\widetilde{\lambda}\pi)\right)^{2j}\left(e^{i\varepsilon(j,j)}e^{2ijX(0)} + e^{i\varepsilon(j,-j)}e^{-2ijX(0)}\right)\right]|0\rangle_c\,. \tag{4.5}$$

It remains to determine the phase factors $e^{i\varepsilon(j,\pm j)}$. These can be determined by using the result [16, 17, 18] that at the special value of $\widetilde{\lambda} = 1/2$, the system becomes a periodic array of D-branes with Dirichlet boundary condition on X, placed at $x = (2n+1)\pi$. Alternatively, at $\widetilde{\lambda} = -1/2$ the system again becomes a periodic array of D-branes, but now placed at $x = 2n\pi$. (In both cases the D-branes are situated at the minimum of the potential $\widetilde{\lambda}\cos(X)$.) Thus for these values of $\widetilde{\lambda}$ the boundary state $|\mathcal{B}\rangle$ must match the known boundary state of the corresponding D-brane system. This gives

$$e^{i\varepsilon(j,\pm j)} = (i)^{2j}\,, \tag{4.6}$$

and hence, writing $j = n/2$, we get

$$|\mathcal{B}_0\rangle \propto \left[1 + 2\sum_{n=1}^{\infty}\left(-\sin(\widetilde{\lambda}\pi)\right)^n \cos(nX(0))\right]|0\rangle_c\,. \tag{4.7}$$

This shows that the source for the closed string tachyon (and other fields involving arbitrary excitations in the rest of the $c = 25$ CFT and the ghost CFT but no oscillator excitation in the $c = 1$ CFT) is proportional to

$$1 + 2\sum_{n=1}^{\infty}\Big(-\sin(\widetilde{\lambda}\pi)\Big)^n \cos(nx)\,. \tag{4.8}$$

We can now make an inverse Wick rotation $x \to -ix^0$ to get the source for the closed string tachyon field (and other fields involving excitations in the $c = 25$ and ghost CFT) for the rolling open string tachyon solution. This is proportional to:

$$f(x^0) \equiv 1 + \sum_{n=1}^{\infty}\Big(-\sin(\widetilde{\lambda}\pi)\Big)^n (e^{nx^0} + e^{-nx^0})\,. \tag{4.9}$$

This sum can be performed explicitly and gives:

$$f(x^0) = \frac{1}{1 + e^{x^0}\sin(\widetilde{\lambda}\pi)} + \frac{1}{1 + e^{-x^0}\sin(\widetilde{\lambda}\pi)} - 1\,. \tag{4.10}$$

Note that for positive $\widetilde{\lambda}$ (which corresponds to beginning on the side on which the potential has a local minimum) the function $f(x^0)$ is finite for all x^0 and in fact goes to 0 as $x^0 \to \infty$. On the other hand if $\widetilde{\lambda}$ is negative, i.e. we begin on the side where the potential is unbounded from below, the function $f(x^0)$ hits a singularity at $x^0 = \ln(-1/\sin(\widetilde{\lambda}\pi))$.

Another interesting quantity is the (Fourier transform of the) coefficient of the state $\alpha^0_{-1}\bar{\alpha}^0_{-1}|k^0\rangle$ in $|\mathcal{B}\rangle$, where α^0_n and $\bar{\alpha}^0_n$ represent the oscillators associated with the right and the left-moving components of X^0. In the Wick rotated picture, this receives contribution from two sources: a constant term proportional to $D^1_{0,0}$ from the primary state $|1;0,0\rangle$, and the secondary states $\frac{1}{2j^2}L_{-1}\bar{L}_{-1}|j;\pm j,\pm j\rangle$ in $|j;\pm j,\pm j\rangle\rangle$. These contributions can be evaluated in a straightforward manner, and after inverse Wick rotation add up to a term proportional to

$$g(x^0) = \cos(2\widetilde{\lambda}\pi) + 1 - f(x^0)\,, \tag{4.11}$$

with $f(x^0)$ defined as in eq. (4.10). Like $f(x^0)$, $g(x^0)$ represents the x^0 dependence of the source for many closed string fields, involving excitation by $\alpha^0_{-1}\bar{\alpha}^0_{-1}$ in the X^0 CFT, and arbitrary excitations in the $c = 25$ CFT and the ghost CFT. In particular the source for different components of the graviton and dilaton fields are given by appropriate linear combinations of $f(x^0)$ and $g(x^0)$.

Since

$$f(x^0) + g(x^0) = \cos(2\widetilde{\lambda}\pi) + 1\,, \tag{4.12}$$

is conserved, it is natural to interprete it as the conserved energy density on the D-brane up to an overall normalization. This interpretation can also be supported by the following observation. For small $\widetilde{\lambda}$, we have

$$f(x^0) + g(x^0) \simeq 2(1 - \widetilde{\lambda}^2\pi^2)\,. \tag{4.13}$$

This should be compared with the initial total energy of the D-brane system when the tachyon field is displaced by a small amount $\lambda \simeq \widetilde{\lambda}$ from its maximum. The tension of the

D-brane is given by $1/(2\pi^2 g^2)$ [5] where g is the open string coupling constant. On the other hand, from the string field theory action we find that for small displacement $\widetilde{\lambda}$ of the tachyon field, the tachyon potential energy is $-\widetilde{\lambda}^2/(2g^2)$. Thus the total initial energy is given by:

$$\frac{1}{2\pi^2 g^2}(1-\widetilde{\lambda}^2\pi^2)\,. \tag{4.14}$$

This is precisely the same as the right-hand side of (4.13) up to a constant of proportionality. This in turn shows that it is correct to interprete $f(x^0)+g(x^0)=1+\cos(2\widetilde{\lambda}\pi)$ as the total energy density of the D-brane system.

We end this section with the following observations:

1. For $\widetilde{\lambda}=1/2$, total energy vanishes. Also from eq. (4.10) we see that $f(x^0)$ vanishes, indicating that the system does not evolve. This corresponds to placing the tachyon at the minimum of its potential. We also note that at this point the Wick rotated theory represents a periodic array of D-branes with Dirichlet boundary condition on the X coordinate, placed at $x=(2n+1)\pi$.

2. For $0<\widetilde{\lambda}<1/2$ the system evolves, but instead of oscillating about the minimum of the potential, its energy momentum tensor asymptotically approaches the configuration $f(x^0)\to 0$ and $g(x^0)\to(1+\cos(2\pi\widetilde{\lambda}))$. It is not clear how to reconcile this result with the result in boundary string field theory that the minimum of the tachyon potential is a finite distance away from the maximum [13, 14].

3. For $-1/2<\widetilde{\lambda}<0$, the system evolves towards a singularity. As $\widetilde{\lambda}$ approaches $-1/2$, the time at which the system hits the singularity goes to zero.

4. The set of inequivalent solutions are obtained by taking $-1/2<\widetilde{\lambda}\le 1/2$. In particular $\widetilde{\lambda}=\frac{1}{2}+\epsilon$ gives the same solution as $\widetilde{\lambda}=\frac{1}{2}-\epsilon$.

5. We have so far analyzed only part of the boundary state. In order to get a complete picture we should also analyze the time evolution of the rest of the boundary state involving higher level states in the X^0 CFT.

6. The analysis can be easily repeated for perturbation by $\widetilde{\lambda}\sinh(X^0)$. In fact this is related to perturbation by $\widetilde{\lambda}\cosh(X^0)$ by a replacement $\widetilde{\lambda}\to -i\widetilde{\lambda}$, $X^0\to X^0+i\pi/2$. Making these replacements in eqs. (4.10) and (4.11) we get

$$f(x^0)=\frac{1}{1+e^{x^0}\sinh(\widetilde{\lambda}\pi)}+\frac{1}{1-e^{-x^0}\sinh(\widetilde{\lambda}\pi)}-1\,, \tag{4.15}$$

and

$$g(x^0)=\cosh(2\widetilde{\lambda}\pi)+1-f(x^0)\,. \tag{4.16}$$

Thus the system hits a singularity at positive (negative) value of x^0 for *small* negative (positive) $\widetilde{\lambda}$. The total energy of the system is now proportional to $1+\cosh(2\widetilde{\lambda}\pi)$. These results are consistent with the interpretation that $\widetilde{\lambda}$ represents the time derivative of the tachyon field at the top of the potential.

5. Effective field theory analysis

In this section we shall use an effective field theory to explore the decay of an unstable brane. The idea behind this analysis is as follows. As pointed out in refs. [24]–[31], the effective field theory describing the coupling of the tachyon to the Born-Infeld lagrangian admits electric flux tube solutions whose dynamics is identical to that of fundamental strings. Thus if we begin with a D-brane with a constant electric field switched on along one of its tangential directions, and then allow the tachyon field to roll down, then we would expect that the end product of the decay process will contain fundamental strings lying along the plane of the original brane, together with other decay products. If the other decay products reside in the degrees of freedom far away from the plane of the brane, we would expect that the hamiltonian describing the dynamics of the electric flux tubes will decouple from the hamiltonian describing the rest of the decay products. On the other hand if the other decay products reside in the plane of the brane, then there will not be any such decoupling.

We shall now show that the decoupling of this type does not occur within the analysis based on an effective field theory, leading to the conclusion that the decay products reside in the plane of the brane. We shall begin with a general effective action on the world-volume of the unstable D-brane, involving a set of scalar fields $\vec{T}$, and a U(1) gauge field A_μ. The scalar fields $\vec{T}$ include the tachyon, and could also include other scalars like the transverse coordinates of the D-brane. We shall restrict to field configurations which are translationally invariant on the brane, work in the $A_0 = 0$ gauge, and allow only A_1 to be non zero, thereby allowing a (time dependent) uniform electric field E along the 1 direction given by

$$E = \partial_0 A_1 \,. \tag{5.1}$$

We shall also make the simplifying assumption that the lagrangian density depends only on the fields $\vec{T}$, A_μ and their first time derivatives. From the general result of [32, 33, 34] it follows that the theory in the presence of the electric field can be related to the theory in the absence of electric field by rescaling the 00 and the 11 components of the metric, changing the overall normalization of the action, and replacing the ordinary products by Moyal product involving the coordinates x^0 and x^1. Since our field configurations are independent of x^1, the Moyal product is the same as the ordinary product, and we only need to take into account the rescaling of the metric and change in the overall normalization of the action. This gives the general form of the lagrangian density for space-independent field configurations to be:

$$\mathcal{L} = \sqrt{1-E^2}\, f\left(\frac{1}{\sqrt{1-E^2}}\,\partial_0\vec{T}, \vec{T}\right), \tag{5.2}$$

where $f(\partial_0\vec{T}, \vec{T})$ denotes the lagrangian density in the absence of the electric field. If we define,

$$f_i(\partial_0\vec{T}, \vec{T}) = \frac{\partial f(\partial_0\vec{T}, \vec{T})}{\partial(\partial_0 T_i)}\,, \tag{5.3}$$

then the momenta Π conjugate to A_1 and the momenta P_i conjugate to T_i are given by, respectively,

$$\begin{aligned}\Pi &= \frac{\partial \mathcal{L}}{\partial E} = -\frac{E}{\sqrt{1-E^2}}\, f\left(\frac{\partial_0 \vec{T}}{\sqrt{1-E^2}}, \vec{T}\right) + \frac{E}{1-E^2}\, \partial_0 T_i\, f_i\left(\frac{\partial_0 \vec{T}}{\sqrt{1-E^2}}, \vec{T}\right), \\ P_i &= \frac{\partial \mathcal{L}}{\partial(\partial_0 T_i)} = f_i\left(\frac{\partial_0 \vec{T}}{\sqrt{1-E^2}}, \vec{T}\right). \end{aligned} \tag{5.4}$$

From this we get

$$\partial_0 T_i = \sqrt{1-E^2}\, g_i(\vec{P}, \vec{T})\,, \tag{5.5}$$

where $\vec{g}$ is the inverse function of $\vec{f}$, i.e.

$$g_i\left(\vec{f}(\partial_0 \vec{T}, \vec{T}), \vec{T}\right) = \partial_0 T_i\,. \tag{5.6}$$

Using eq. (5.5), we can eliminate $\partial_0 T_i$ in terms of P_i in the first of eq. (5.4) and get

$$\Pi = \frac{E}{\sqrt{1-E^2}}\, h(\vec{P}, \vec{T})\,, \tag{5.7}$$

where

$$h(\vec{P}, \vec{T}) = \left(-f\left(\vec{g}(\vec{P}, \vec{T}), \vec{T}\right) + \vec{P} \cdot \vec{g}(\vec{P}, \vec{T})\right). \tag{5.8}$$

This gives

$$E = \frac{\Pi}{\sqrt{\Pi^2 + \left(h(\vec{P}, \vec{T})\right)^2}}\,. \tag{5.9}$$

Let us now turn to the construction of the hamiltonian density. This is given by:

$$\mathcal{H}(\vec{P}, \vec{T}) = \Pi\, E + \vec{P} \cdot \partial_0 \vec{T} - \mathcal{L} = \sqrt{\Pi^2 + \left(h(\vec{P}, \vec{T})\right)^2}\,. \tag{5.10}$$

This shows that $h(\vec{P}, \vec{T})$ represents the hamiltonian density of the system for $E = 0$. $\mathcal{H}$ is conserved as a result of time translation invariance of the system, and Π is conserved as a result of gauge invariance. Thus from (5.10) we see that $h(\vec{P}, \vec{T})$ is also conserved, and eq. (5.9) then shows that E is conserved as well. We note from eq. (5.10) that due to separate conservation of Π and h, the dynamics of $\vec{P}$ and $\vec{T}$ in the background of electric flux is related to the dynamics in the absence of the flux by a simple time dilation factor of $h/\sqrt{\Pi^2 + h^2}$.

Π can be interpreted as the energy stored in the electric flux tube; indeed if during the evolution of the D-brane the excess energy leaves the plane of the brane leaving behind static electric flux tubes in the tachyon vacuum, then the energy stored in the flux tube will be exactly Π [27, 29, 31]. In this case we would expect the hamiltonian describing the dynamics of the flux tube to be nearly decoupled from the hamiltonian describing the dynamics of the "bulk modes". As seen from (5.10) however, this is not the case; the hamiltonian $h(\vec{P}, \vec{T})$ containing the excess energy of the brane couples strongly to Π. To see this more explicitly, we can compare two situations. First consider the situation where we have an initial condition $h(\vec{P}, \vec{T}) = 0$. This represents a system with electric flux at the minimum of the tachyon potential. In the other case we begin with a system with the same amount of electric flux near the top of the potential well and let it roll down towards the minimum, so that the initial condition sets h to a fixed non-zero constant required to give the correct tension of the brane. If the excess energy of the brane is carried away from the brane during the decay leaving behind static electric flux tubes, then after a sufficiently long time, the dynamics of the flux tube in the two cases should be described effectively by the same hamiltonian. To see if this is the case, we now compare the response of these two systems to small fluctuation of the scalar fields X^i describing transverse coordinates of the brane. This can be done by deforming h from its given value by switching on small amount of momenta P_i conjugate to X^i. Physically, around the tachyon vacuum solution, P_i represents the momentum carried by the flux tube in directions transverse to the original brane [29, 31]. Since $h(\vec{P}, \vec{T})$ is conserved, irrespective of how long we wait, the dependence of $\mathcal{H}$ on P_i will be different in the two cases, since we are Taylor expanding a function around two different points. Thus the dynamics of the flux tube continues to depend on the initial energy of the system irrespective of how long we wait. From this we conclude that the excess energy does not get carried away by radiation, but resides in the plane of the brane where it couples strongly to the electric flux tubes.

6. Conclusion

In this paper we have explored some aspects of time dependent solution in string field theory on an unstable D-brane describing the tachyon rolling away from the maximum of its potential. Due to the presence of higher derivative terms in the string field theory action it is not *a priori* clear how to construct such solutions. We have shown that it is indeed possible to construct a family of solutions of the string field theory equations of motion labelled by the initial position and velocity of the tachyon field. Furthermore, one can also explicitly write down the boundary conformal field theory action and the boundary state associated with these solutions.

Unlike in the case of decay of an unstable lump solution in a generic field theory, where under a small perturbation the energy of the lump gets converted to the classical radiation moving away from the original position of the lump, in the case of classical decay of a D-brane, the energy is stored in the plane of the original brane. This can be traced to the fact that there are no open string degrees of freedom away from the plane of the D-brane, and hence no classical mode of the string can carry away the energy.

The analysis of the boundary state shows that for D-branes in bosonic string theory, if we push the system towards the minimum of the tachyon potential, its energy momentum tensor evolves smoothly towards an asymptotic configuration. Instead, if we push the system in the other direction in which the potential is unbounded from below, the energy momentum tensor quickly hits a singularity.

Clearly there are many avenues left open for exploration. This includes a detailed study of the boundary states describing the rolling of the tachyon field on unstable D-branes in superstring theories. In the Wick rotated theory this was studied in ref. [35], but since it was expressed in terms of a different set of variables than the Wick rotated coordinate X, it is difficult to use it directly for the present analysis. Another possible approach would be construction of the time dependent solutions in vacuum string field theory [36] along the line of [37]. These studies may turn out to be useful in uncovering the role of tachyon condensation in cosmology.

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